Distributed Detection in Cognitive Radio Networks with Unknown Primary User's Traffic

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***Abstract*—We consider a distributed Cognitive Radio Network (CRN) in which a number of secondary users (SUs) cooperatively sense the spectrum holes in a channel whose occupancy by the primary user (PU) follows a discrete time Markov chain. We consider a problem where the transition probability matrix (TPM) of the channel occupancy process followed by the PU is unknown. For this problem, we first propose a procedure to estimate the TPM. We then seek a distributed detection procedure with the estimated TPM. We show that our estimate of the TPM is very close to that of the actual TPM in the case of perfect sensing; in the case of imperfect sensing, the estimate has a little bias. This bias can be used to correct the estimate further. We study the detection and the false alarm characteristics of the distributed detection procedure that we propose (which uses the estimates on the channel occupancy probabilities). The proposed distributed detection procedure achieves a good Receiver Operating Characteristic (ROC) performance which is close to that of the centralised CRN.**

I. Introduction

Due to rapid growth in wireless technologies, the spectrum is in high-demand, and hence, there is a need to increase the spectral efficiency. This necessitates the exploration of spectrum usage. On the other hand, studies show that the spectrum allocated to licensed users is heavily underutilized.

Cognitive Radio is proposed as a means to exploit the spectrum vacancies through Opportunistic Spectrum Access (OSA). In OSA, the channel allocated to a primary user (PU), also called licensed user, is sensed by secondary users (SUs) for detecting the signal transmission of the PU, and obtain observations regarding the channel occupancy. Using the observations, SUs detect the availability of a channel, and the possible access. In spectrum sensing and in channel access by the SU, the primary objective is that no interference is caused to the PUs by any SU.

A. Previous Work

In [1] and [2], the authors propose myopic sensing policies for multichannel OSA. Here, the SU choose channels to sense and access based on their beliefs about the channel states at each time-slot. The work assumes complete knowledge of the distribution of traffic of PUs, and this is used to compute the belief of channel occupancy.

In [3], [4], [5], and [6] decentralized strategies to decide which channel to sense and access with an objective of maximising a network throughput are proposed. The papers propose detection procedures based on belief vector as a sufficient statistic. Optimal access strategy is also described for a centralized decision-making problem.

In [7], the authors describe a learning procedure to infer the unknown distribution of the traffic of PU is obtained. Here, the lack of knowledge of received signal power is modeled by assuming an unknown parameter for the mean of observations. The channel to be sensed at the beginning of any time-slot depends on the set of beliefs of channel occupancy of each channel. This procedure requires a reliable model for the state transition process.

In [8], a heuristic approach is proposed to exploit the channel correlation in sensing; the throughput degradation observed when channel correlation is ignored is also discussed.

[9] discusses a fusion rule of local observation based on a linear combination of the local statistic. This weighted linear combination is transmitted to a Fusion Center (FC) through a control channel. This necessitates the requirement that the reporting channel bandwidth be at least the same as the bandwidth of the sensed channel.

Almost all the literature work has a central entity, which is the FC. We note that the FC receives all local decisions from each of the SUs, and declares a global decision. A centralized Cognitive Radio Network (CRN), though has a better performance than a distributed network, is not feasible in the case of ad-hoc networks. Also, in a centralized CRN, the decision about the channel availability and its access is fed back to the SUs, which requires a high reliable feedback channel.

In the case of wireless CRN, fading and shadowing effects make the sensing of individual SUs highly unreliable. It has been shown that cooperation among SUs increases the sensing and detection performance. In particular, when the traffic of the PU (whether there is a packet to transmit, or not) follows a discrete-time Markov chain (DTMC), the past observations of the SU about the channel occupancy of PU can be used to obtain a belief vector which is used to propose a detection policy. This belief vector is simply a posterior probability of the channel being idle given all observation of an SU till the present instant.

In this paper, we consider a distributed cooperative spectrum

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sensing (CSS) method. In distributed CSS, the local decisions of all SUs is shared amongst them, and a global decision is then computed by each SU. Our work considers a practical scenario where the SUs in a distributed CRN are unaware of the state transition probabilities (STPs) of the channel occupancy process of the PU. We propose a method to estimate the STPs based on the past observations of PU. We then use the estimates of the STPs to obtain a distributed detection procedure.

B. Contributions of the Paper

We summarise the important contributions of the paper below:

- We propose a distributed procedure for estimating the STPs of the channel occupancy process of the PU in a CRN.
- We propose a local detection procedure at each SU based on the a posteriori probability that the channel is idle given all observations until the current time-slot. These local decisions are combined in a weighted linear manner, and a global decision is obtained.
- A new approach to estimate the probabilities is provided using a reward function, which assigns a reward to an SU based on whether it's local decision is in agreement with the global decision. A counting function is used to compute the STPs which is shared intermittently.
- The detection problem when the information is shared in every nth slot is discussed. This discussion will help in finding how often should SUs share their information for optimal detection.

C. Organization of the paper

The rest of the paper is organised as follows. In Section II, we describe the the model of the CRN, the sensing model of the SUs, and the traffic model of the PU. In Section III, we discuss a distributed procedure to estimate the unknown parameters of the traffic. Also, we propose a distributed procedure for channel sensing and access. In Section IV, we discuss the simulation results and the inference from the results. Conclusions and future work are discussed in Section V.

II. System Model

In this Section, we define the system model for the CRN and the sensing model with which each SU obtains it's observations.

A. Network Model

We consider a cognitive radio network with one PU (and hence one channel) and L SUs. The set of L SUs are denoted by $\mathcal{S} = \{1, 2, \cdots, L\}$. Time is slotted, and the length of each time-slot is assumed to be T. Time-slots are indexed by $k \in \{1, 2, 3, \cdots\}$. In each time-slot k, the PU is either having a packet to transmit (which we call busy state), or not (which we call idle state). The state of the PU is denoted by $\Theta[k] \in \{0, 1\}$, where 0 indicates busy, and 1 indicates idle.

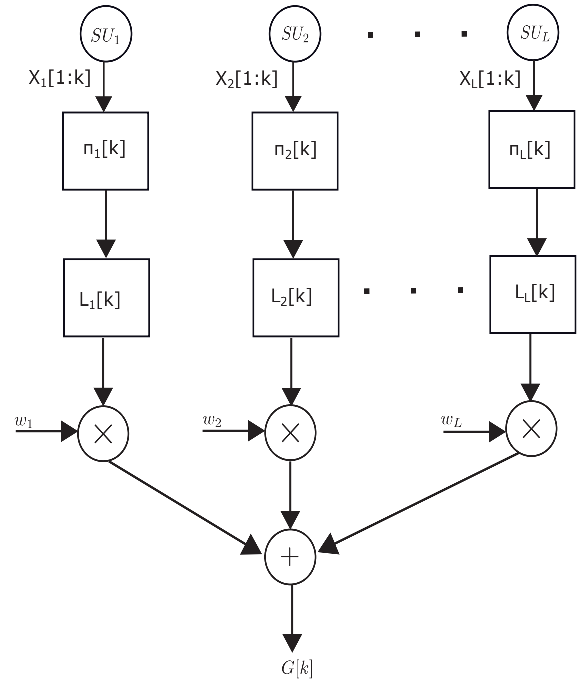

Fig. 1. System for Distributed Detection when the TPM $\mathbf{P}$ is unknown

We consider the state of the PU (i.e., $\{\Theta[k] : k \in \{1, 2, \cdots\}\}$) to follow a discrete-time Markov chain (DTMC) with the following transition probability matrix (TPM),

$$\mathbf{P} = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}. \tag{1}$$

In this paper, we consider a detection problem when the SUs know that the state of the PU follows a DTMC, but the TPM $\mathbf{P}$ is not known.

B. Sensing Model

At the beginning of time-slot k, let $X_s[k] \in \{0, 1\}$ be the observation of SU s about the state of PU. We consider a non-ideal sensing model, in which the state of the PU that is sensed by the SU is possibly erroneous. The sensing model that we consider in this work is as follows.

$$\begin{aligned} &\mathbb{P}\{X_s[k] = 1 \mid \Theta[k] = 0\} = \alpha_s, \\ \text{and } &\mathbb{P}\{X_s[k] = 1 \mid \Theta[k] = 1\} = \beta_s. \end{aligned} \tag{2}$$

Note that

$$\begin{aligned} &\mathbb{P}\{X_s[k] = 0 \mid \Theta[k] = 0\} = 1 - \alpha_s, \\ \text{and } &\mathbb{P}\{X_s[k] = 0 \mid \Theta[k] = 1\} = 1 - \beta_s. \end{aligned} \tag{3}$$

Also, we note here that α_s and β_s define the sensing quality of each SU s. However, for the sake of convenience, we keep $\alpha_s = \alpha$ and $\beta_s = \beta$ for all SUs.

In the next Section, we discuss a detection statistic, local decision rule, and a global decision rule for channel sensing and access.

$$
\Pi_s[k+1] = \begin{cases} \dfrac{\left[\left(1-\Pi_s[k]\right)p_{01}+\Pi_s[k]p_{11}\right](1-\beta)}{\left[\left(1-\Pi_s[k]\right)p_{00}+\Pi_s[k]p_{10}\right](1-\alpha)+\left[\left(1-\Pi_s[k]\right)p_{01}+\Pi_s[k]p_{11}\right](1-\beta)}, & \text{if } X_s[k+1]=0, \\ \dfrac{\left[\left(1-\Pi_s[k]\right)p_{01}+\Pi_s[k]p_{11}\right]\beta}{\left[\left(1-\Pi_s[k]\right)p_{00}+\Pi_s[k]p_{10}\right]\alpha+\left[\left(1-\Pi_s[k]\right)p_{01}+\Pi_s[k]p_{11}\right]\beta}, & \text{if } X_s[k+1]=1. \end{cases} \tag{4}
$$

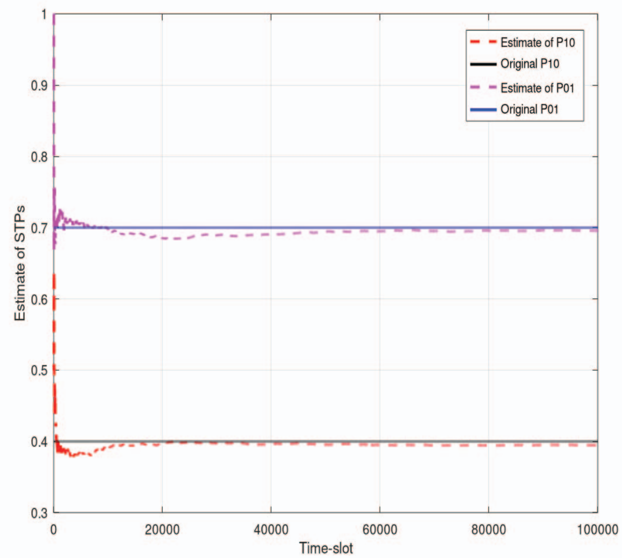

Fig. 2. Estimated transition probabilities against number of time slots for the case of perfect sensing. The experiment has been conducted with the following parameters: $L = 5$, number of time-slots = 10^4, $\alpha_s = 0, \beta_s = 1, p_{01} = 0.7, p_{10} = 0.4, \gamma = 0.5, \Gamma = 0.5$.

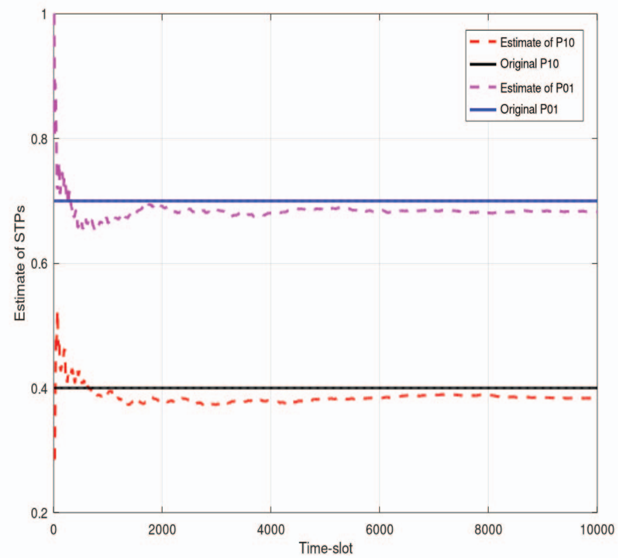

Fig. 3. Estimated transition probabilities against number of time slots for the case of imperfect sensing. The experiment has been conducted with the following parameters: $L = 5$, number of time-slots = 10^4, $\alpha_s = 0.8, \beta_s = 0.2, p_{01} = 0.7, p_{10} = 0.4, \gamma = 0.5, \Gamma = 0.5$.

III. Distributed Detection with Unknown Transition Probability Matrix

At the beginning of each time-slot k, each SU s senses and obtains an observation $X_s[k]$. It is well known that for a centralized detection problem (or, for a single sensor problem), a sufficient statistic for detection is the a posteriori probability that the PU is idle. Let $\Pi_s[k]$ denote the a posteriori probability that the PU is idle during slot k given the sensing data of SU s until time-slot k, i.e.,

$$
\Pi_s[k] := \mathbb{P}\{\Theta[k] = 1 \mid X_s[1:k]\}, \tag{5}
$$

where the notation $X_s[1:k]$ indicates the vector of sensing data $[X_s[1], X_s[2], \cdots, X_s[k]]$. We note that from $\Pi_s[k]$ and $X_s[k+1]$, we can compute $\Pi_s[k+1]$, which we provide in Eqn. (4).

Since the TPM $\mathbf{P}$ is not known to the SUs, we compute p_{ij}s of the TPM in an iterative manner. Let $\hat{p}_{ij}[k]$ be the estimate of p_{ij} by SU s at the beginning of time-slot k. $\hat{p}_{ij}[k]$s are updated at the end of each time-slot k which is explained at the end of this Section.

Each SU s, using the estimate $\hat{p}_{ij}[k]$ of the TPM, computes the a posteriori probability $\Pi_s[k]$ (refer Eqn. (5)). SU s then makes a local decision of the state of PU during time-slot k, as follows,

$$
L_s[k] = \begin{cases} 1, & \text{if } \Pi_s[k] \geq \gamma, \\ 0, & \text{if } \Pi_s[k] < \gamma, \end{cases} \tag{6}
$$

where the threshold $\gamma \in (0,1)$.

Each SU s broadcasts its local decision $L_s[k]$ to all other SUs in each slot k. Thus, each SU s has the local decisions of all other SUs, and computes a metric which is the sum of weighted local decisions, which is defined as

$$
D_s[k] = \sum_{s=1}^{L} w_s[k] \cdot L_s[k], \tag{7}
$$

where $w_s[k]$ is the weight assigned to the local decision of SU s during time-slot k.

Each SU s makes a global decision $G[k]$, which is defined as

$$
G[k] = \begin{cases} 1, & \text{if } D_s[k] \geq \Gamma, \\ 0, & \text{if } D_s[k] < \Gamma. \end{cases} \tag{8}
$$

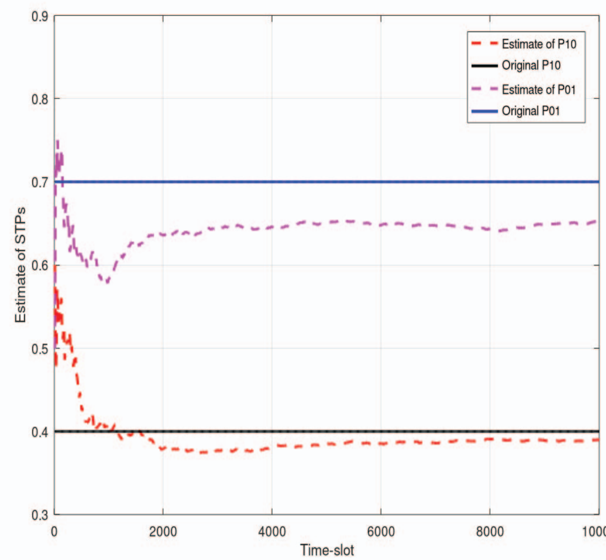

Fig. 4. Trial 1: Estimated transition probabilities against number of time slots for the case of missing data in every alternate slot for a realization of Markov chain modeled traffic pattern of PU. The experiment has been conducted with the following parameters: $L = 5$, number of time-slots = 10^4, $\alpha_s = 0.8, \beta_s = 0.2, p_{01} = 0.7, p_{10} = 0.4, \gamma = 0.5, \Gamma = 0.5$.

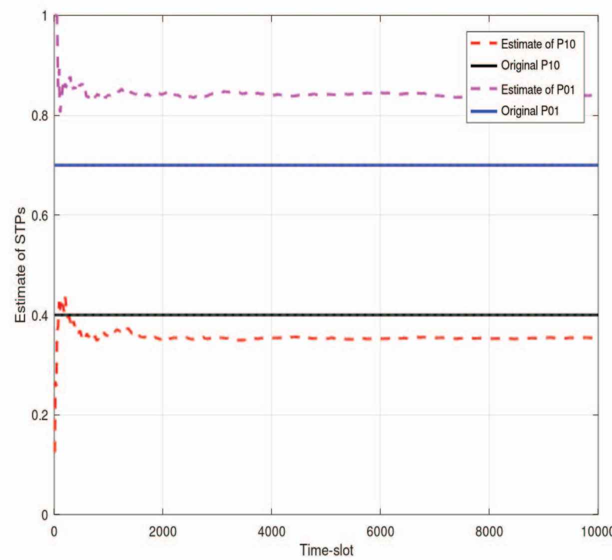

Fig. 5. Trial 2: Estimated transition probabilities against number of time slots for the case of missing data in every alternate slot for a different realization of Markov chain modeled traffic pattern of PU. The experiment has been conducted with the following parameters: $L = 5$, number of time-slots = 10^4, $\alpha_s = 0.8, \beta_s = 0.2, p_{01} = 0.7, p_{10} = 0.4, \gamma = 0.5, \Gamma = 0.5$.

A. Rule for updating $w_s[k]$

With each SU s, we associate a reward $R_s[k]$ during time-slot k, which is updated at the end of each time-slot. We set

$$w_s[k] = \frac{R_s[k-1]}{\sum_{s=1}^{L} R_s[k-1]}. \tag{9}$$

At the end of each time-slot k, each SU s gets a reward as follows,

$$R_s[k] = \begin{cases} 1, & \text{if } L_s[k] = G[k], \\ -1, & \text{if } L_s[k] \neq G[k]. \end{cases} \tag{10}$$

B. Rule for updating $\hat{p}_{ij}[k]$s

At the beginning of each time-slot k, let $C_{ij}^{(s)}[k]$ be the number of state transitions of PU from state i to state j until time-slot k. Each SU s broadcasts $C_{ij}^{(s)}[k]$ in every nth slot, and choose the $C_{ij}^{(s^*)}[k]$ of an SU s^* which has maximum reward. The transition probability $\hat{p}_{ij}[k]$ can be computed as follows

$$\hat{p}_{ij}[k] = \frac{C_{ij}^{(s^*)}[k]}{C_{ij}^{(s^*)}[k] + C_{ij'}^{(s^*)}[k]}, \tag{11}$$

where $j' = 0$ if $j = 1$, and $j' = 1$ if $j = 0$. We keep the same $\hat{p}_{ij}[k]$ between successive updates on TPM.

IV. Results and Discussion

In this Section, we describe the setting of the CRN, that we use to obtain the numerical results for the distributed procedure we explained in the previous Section.

We first evaluate our procedure of estimating the transition probabilities (see Eqn. (11)). We consider the following three cases: i) when perfect sensing is assumed (i.e., $\alpha_s = 0$ and $\beta_s = 1$), ii) when the SUs have noisy observations of the channel obtained from measurements (i.e., $\alpha_s > 0$ and $\beta_s < 1$), and iii) when sensing observation is not available in every slot. We evaluate the estimation procedure for a CRN having 1 PU and L = 5 SUs. The experiments are designed for 10^4 time-slots. Updating $\hat{p}_{ij}[k]$s takes place every nth slot where $n = 10$. For each time-slot, the threshold applied on $D[k]$ is $\Gamma = 0.5$, and the threshold applied on $\Pi_j[k]$ is $\gamma = 0.5$.

Figs. 2 and 3 show the evolution of $\hat{p}_{ij}[k]$ over time-slots for the cases (i) perfect sensing, and (ii) imperfect sensing respectively. We can see that a steady state value for $\hat{p}_{ij}[k]$ is achieved after 40000 time-slots for perfect sensing. A biased but consistent estimate is observed for the noisy observation case.

For the case of missing data (as in case iii), a variability is observed in the estimated $\hat{p}_{ij}[k]$s for every realization of the Markov chain modeled traffic of PU. As shown in Fig. 4 and Fig. 5 for two such realizations represented by trial 1 and trial 2 respectively, the first realization has $\hat{p}_{01}$ reaching a steady value of 0.65 while a second trial has the estimate nearing the value 0.9. In both the realizations, $\hat{p}_{10}$ approximates a

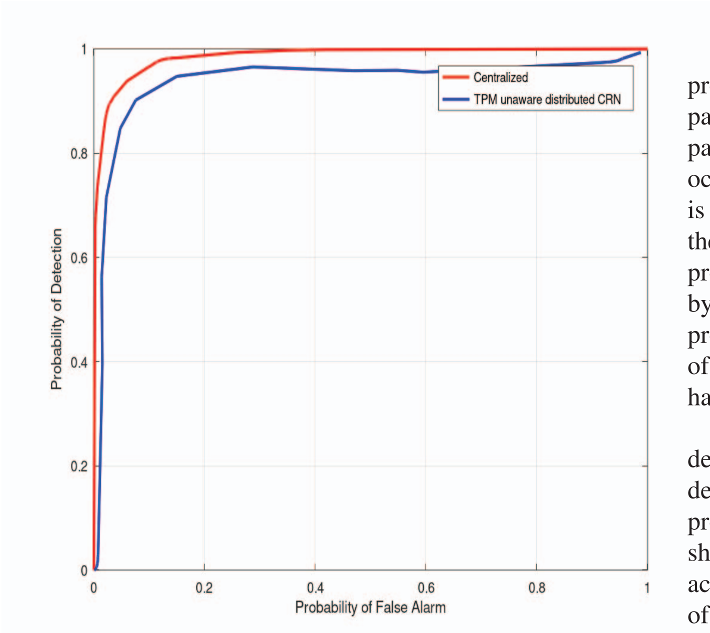

Fig. 6. A comparison of the ROC curve for distributed CRN with unknown STPs and centralized network. The experiment has been conducted with the following parameters: $L = 5$, number of time-slots = 10^4, $\alpha_s = 0.8, \beta_s = 0.2, \gamma = 0.5$.

value close to 0.4 without a large variance as observed in the case of $\hat{p}_{10}$. Hence for high transition probability like $p_{01} = 0.7$, a range of values from [0.65 0.9] can be observed. A lower transition probability like $p_{10} = 0.4$ shows a biased but consistent estimate in every realization. The variance of the estimate depends on the number of missing slots.

This is because as transition probability (e.g., p_{01}) increases, our algorithm involves counting the state transitions which will now be higher. Since the observations are noisy and missing, there is more probability of error seen in estimation of p_{01} as we now predict the missing slots too. Hence, the algorithm approximates the true transition probability in the cases where the STPs are not high, subsequently making it less prone to errors.

We evaluate the probability of detection and the probability of false alarm performance of the distributed procedure for sensing and channel access for a CRN having 1 PU and L = 5 SUs. We plot the results in Fig. 6, which shows the Receiver Operating Characteristic (ROC) curve of the proposed detection procedure for various values of the global threshold Γ. For each of the global threshold Γ, we run the experiment for 10^4 time–slots, and obtain the probability of detection and the probability of false alarm. It is evident from Fig. 6 that our procedure based on weighted linear combination achieves a good performance, i.e., for a given probability of false alarm, probability of detection of our procedure is large. Also the ROC curve is very close to that of centralized network which always achieves the best performance.

V. Conclusions

In this paper, we have considered a distributed detection problem to detect the availability of the channel when the parameters of the PU's traffic are not known to the SUs. In particular, we have considered the problem where the channel occupancy of the PU follows a DTMC, the TPM of which is unknown to the SUs. We provide a procedure to estimate the TPM of the traffic statistics of the PU. Our estimation procedure is based on a mechanism of rewards obtained by each SU. We have shown that the proposed detection procedure achieves a slightly biased but good approximation of the transition probabilities of the PU, even when the SU has noisy observations.

We then propose a global decision rule that makes a global decision when the weighted linear combination of the local decisions crosses a global threshold Γ. We have compared our procedure with a centralized network of known TPM, and have shown that our procedure performs close to the centralised and achieves a good probability of detection for a given probability of false alarm.

We have also studied the effect of intermittent sharing of observation of measurements by the SUs. Our results show that our distributed procedure is very close to the ROC performance of the centralized CRN.

Our future work lies in reducing the variance observed in estimation of STPs for the case of missing data in certain time-slots. The work on distributed detection procedure can be extended to analyse the detection and false alarm probabilities, and the throughput obtained.

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