Minimum Age Scheduling in Fading Channels With Delayed CSIT and Energy Constraints

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***Abstract*—We study a problem of scheduling transmissions of status messages over a fading link. The fading process is i.i.d., and is known at the sender after one-slot delay. Messages arrive according to an i.i.d. Bernoulli process at the sender, and each message has a delay deadline of *d* time-slots. The problem is to decide whether or not to transmit, and to choose an optimum transmit-energy, such that the average Age of Information is minimum satisfying a constraint on transmit-energy and throughput. We pose this problem as a constrained Markov decision process, and provide an Age OPTimum policy (AOPT). We also propose a simple heuristic, Greedy Randomization of Energy Policy (GREP), and compare the performance of AOPT and GREP with ARQ policies. This is one of the first work in finding a minimum age scheduling policy with throughput guarantees and with delayed CSIT for an average transmit-energy constraint.**

***Index Terms*—Average age of information (AAoI), automatic repeat request (ARQ), constrained Markov decision process (CMDP), delay deadline, delayed CSIT, hybrid ARQ (HARQ).**

I. Introduction

In INTERNET of Things (IoT) systems, timely updates from sensors play a very prominent role in control and automation. The time-stamp of sensor messages determines information freshness, defined as Age of Information (AoI) [1]. In this Letter, we consider a source that sends messages to a sink through a fading channel. The source has a transmit-energy constraint, and the channel state information (CSI) at the transmitter (CSIT) is unknown at the transmission instant. Hence, a fundamental question is *how to schedule packet transmissions to achieve a minimum AoI when the current fading state is unknown and the average transmit-energy is bounded.* In multisensor IoT/cyber-physical systems, it is important to make an optimal decision based on a sufficient number of samples received from each source. Also, in news bots, previous/old messages are very important for a deeper insight of an event (e.g., in an epidemic outbreak, to infer source of outbreak, identifying hotspots, etc.). Thus, it is important to have throughput guarantees, as well with data freshness.

A. Previous Work

Queues with service errors are analyzed for AoI under standard automatic repeat request (SARQ) protocols in [2]. Scheduling transmissions from a single source that generates messages at will with unknown channel errors has been studied in [3]–[5]. [3] explores energy-delay tradeoff for SARQ. In [4], hybrid ARQ (HARQ) with chase combining (CC-HARQ) is studied with a constraint on the average number of transmissions. However, [4] does not consider dynamic links, or control on transmit-energy. [3], [4] rely on ACK/NACK feedback for control policy. With sampling cost, a joint optimal scheduler for sampling and updating for a single IoT source is studied [5]. For a single source, single buffer systems without feedback, truncated ARQ (TARQ) [6], [7] retransmits a message a fixed number of times, or until a new message arrives. In [8], scheduling packets with delay deadlines[1] through an FCFS M/M/1/k queue is studied. It is shown that for large arrival rates, as delay deadline decreases, AAoI decreases. However, [6]–[8] use same power for all transmissions. Thus, a *scheduler that minimizes AAoI of a delay-sensitive system with delayed CSIT with average transmit-energy and throughput constraints* has not been studied, which we seek in this Letter.

B. Contributions of This Letter

The contributions of this letter are given below.

1) We solve minimum age scheduling problem with delayed CSIT, and energy and throughput constraints, with packet delay deadlines. We obtain an Age OPTimum policy (AOPT) for FCFS and LCFS service disciplines.
2) We show that the structure of AOPT is a mixture of two stationary deterministic policies. Based on the structure, we propose a computationally simple policy, Greedy Randomized Energy Policy (GREP), the AAoI performance of which is very close to that of AOPT.
3) We show the tradeoff between throughput and AAoI as delay deadline[1] d, increases.

II. System Model

A. Network Model

We consider a source that sends messages (a.k.a. packets) to a sink through a fading link. Time is measured in discrete time-slots, $t \in \mathbb{Z}_+ := \{0, 1, 2, \ldots\}$, the length of which is a packet transmission time. At each time-slot t, the source generates $A[t] \in \{0, 1\}$ packets according to an independent and identically distributed (i.i.d.) Bernoulli(p) process.

Transmit-Queue: A packet upon arrival is stored in a transmit-buffer, and waits for transmission in FCFS or LCFS

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[1]A delay deadline of d defines the maximum delay that a packet can have before which it has to be delivered; otherwise, if the delay is more than d, the packet is dropped at the source. This is further explained in Section II-A.

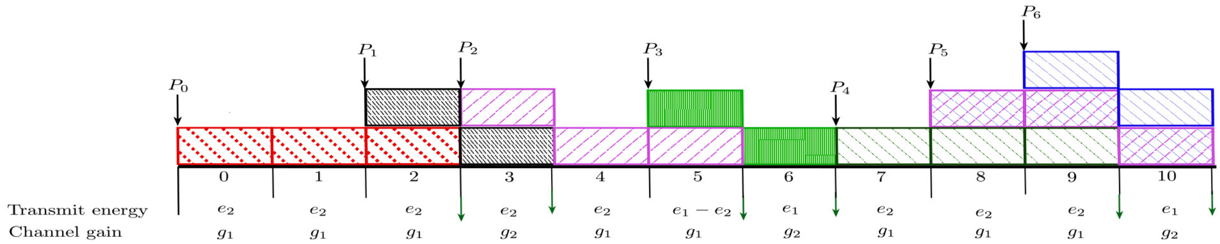

Fig. 1. An illustration of scheduling for a delay deadline of $d = 2$ slots. Packets P_0, P_1, P_2, P_3, P_4, and P_5 arrive at time-slots 0, 2, 3, 5, 7, and 8, and depart at time-slots 2, 3, 5, 6, 9, and 10, respectively. Retransmissions of packets P_0, P_2, P_4 are combined at the sink by a chase combiner. In time-slot 3, e_2 is just enough for channel gain g_2, whereas in time-slot 10, e_1 is more than required for channel gain g_2, and hence, packets P_1 and P_5 are received successfully. In time-slot 5, at least $e_1 - e_2$ is required; otherwise, P_2 is dropped. *This example illustrates the significance of $e_1 - e_2$ under delayed CSIT.*

discipline until its waiting time exceeds a delay deadline of d time-slots,[2] or is successfully delivered.

At time-slot t, let $Q[t]$ be the number of packets waiting for transmission in the transmit-buffer. The packets in the buffer are numbered $1, 2, \ldots, Q[t]$ in the order of arrival, with 1 being the head of line (HoL), and $Q[t]$ being the tail of line (ToL) packet. For FCFS, HoL packet is transmitted, and for LCFS discipline, ToL packet is transmitted. Let the waiting time of packet i at time-slot t be $W_i[t]$. Define $\boldsymbol{W}[t] = [W_1[t], W_2[t], \ldots, W_{Q[t]}[t]]$. Let $X[t+1] = 1$ denote a successful delivery in time-slot t; $X[t+1] = 0$ denotes no success. The number of departures in time-slot t is denoted by $D[t+1] \in \{0, 1, 2\}$, where a departure may be due to success, or due to drop (if $W_1[t] = d$). $D[t+1] = 2$ happens only in LCFS if $Q[t] > 1$, $X[t+1] = 1$, and $W_1[t] = d$. Thus,

$$Q[t+1] = Q[t] - D[t+1] + A[t+1]. \tag{1}$$

If $Q[t] - D[t+1] > 0$, at time-slot $t+1$, waiting time of packets $i = 1, 2, \ldots, Q[t] - D[t+1]$ is

$$W_i[t+1] = \begin{cases} W_{i+1}[t] + 1, & \text{if FCFS, } X[t+1] = 1, \text{ or } W_1[t] = d, \\ W_{i+1}[t] + 1, & \text{if LCFS, } W_1[t] = d, \\ W_i[t] + 1, & \text{otherwise,} \end{cases} \tag{2}$$

and if there is a new arrival, $W_{Q[t+1]}[t+1] = 0$.

B. Transmission Model

Channel model: We consider a finite-state flat fading channel with power gain (or state) $G[t] \in \mathbb{G} = \{g_1, g_2, \ldots, g_K\}$ in time-slot t, where $g_1 < g_2 < \cdots < g_K$. $\{G[t] : t \in \mathbb{Z}_+\}$s are i.i.d. with probability mass function $p_G(\cdot)$. An instantaneous CSI is available at the sink, whereas only one-slot delayed CSI is available at the transmitter. Let the CSIT at time-slot t be $\tilde{G}[t]$, where $\tilde{G}[t] = G[t-1]$.

Communication model: We consider an SNR model for reliable communication that requires an SNR of at least γ. For a noise variance $N_0/2$, the minimum energy required to achieve an SNR of γ in fading state g_i in one ARQ round is $e_i = (N_0/2)\gamma/g_i$, Note that $e_1 > e_2 > \cdots > e_K$. Let $U[t]$ be the transmit-energy chosen at time-slot t. We require $U[t] = \frac{N_0/2}{G[t]}\gamma$, but the sender knows $G[t]$ only at time-slot $t+1$. Thus, it may happen that the chosen transmit-energy $U[t]$ may result in an SNR $< \gamma$, requiring at least one more round of HARQ. A chase combiner combines all transmissions of a packet (see Fig. 1).

If the transmit-queue is non-empty, the controller makes a decision either to transmit a packet or not. Note that $U[t] \in \{0, e_1, e_2, \ldots, e_K\} =: \mathbb{U}_0$. For retransmission, $U[t]$ is chosen from $\mathbb{U} = \mathbb{U}_0 \cup \{e_i - e_j : 1 \leq i < j \leq K\}$. Let $R[t]$ denote the sum of SNRs of all previous transmissions of the current packet at time-slot t, and $Y[t] := U[t]G[t]/(N_0/2)$ denote the SNR of the transmission during time-slot t. A packet is successfully delivered only if the sum of SNRs, $R[t] + Y[t] \geq \gamma$. Thus,

$$X[t+1] = \begin{cases} 1, & \text{if } R[t] + Y[t] \geq \gamma, \\ 0, & \text{otherwise, and} \end{cases}$$

$$R[t+1] = \begin{cases} (R[t] + Y[t])\mathbf{1}_{\{X[t+1]=0\}}, & \text{if FCFS,} \\ (R[t] + Y[t])\mathbf{1}_{\{X[t+1]=0, A[t+1]=0\}}, & \text{if LCFS.} \end{cases} \tag{3}$$

In LCFS, a new arrival erases all copies of unsuccessful transmissions of previous packets, if any, at the sink.

C. Age of Information (AoI)

The information freshness at the sink is measured by Age of Information (AoI) [1], which is defined as time elapsed since the generation of the freshest packet delivered at the sink. Let the packets be labeled $\{1, 2, \ldots\}$ such that their arrival epochs $Z_1, Z_2, \ldots,$ are ordered (i.e., $Z_1 < Z_2 < \cdots$). At time-slot t, let $m(t) \in \{1, 2, \ldots\}$ be the freshest packet received at the sink. Thus, AoI denoted by $H[t]$, is defined as[3]

$$H[t] := (t - Z_{m(t)})_M, \tag{4}$$

where M (saturation age) is the maximum delay beyond which the received packet is stale. Thus, an AoI of M means that the freshest packet received is generated at least M time-slots earlier. The AoI defined in (4) helps in keeping the set of ages finite. The evolution of AoI at time-slot $t+1$ is

$$H[t+1] = \begin{cases} (H[t]+1)_M, & \text{if } X[t+1] = 0, \\ \min\{H[t]+1, (W_i[t]+1)_M\}, & \text{if } X[t+1] = 1, \end{cases}$$

where $W_i[t] := W_1[t]$ for FCFS, and $W_{Q[t]}[t]$ for LCFS. AoI increments with t until a freshest packet is received

[2]For a delay deadline requirement of d, a packet that arrives at time-slot t is required to be delivered at the sink before the end of time-slot $t + d$; otherwise, at the beginning of time-slot $t + d + 1$, the packet is dropped.

[3]We define the following notation: for any integers x and M,

$$(x)_M := \min\{M, x\}.$$

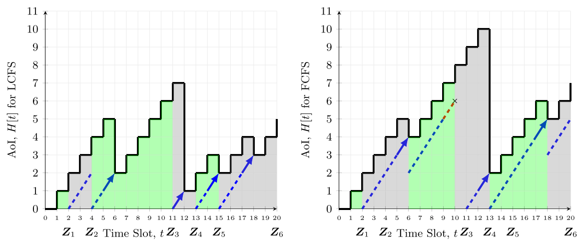

Fig. 2. Evolution of AoI for LCFS (left) and FCFS (right) disciplines.

successfully at t', at which point, $H[t'] = (t' - Z_{m(t')})_M$. Note that $\forall t < t'$, $m(t) < m(t')$, and at t', $H[t']$ is the waiting time of the freshest packet received (see Fig. 2).

D. Throughput

The long-term time average throughput of the system is

$$\eta = \lim_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\sum_{t=0}^{T-1} X[t+1]\right]. \quad (5)$$

A large transmit-energy yield a high throughput. Our interest is to design a minimum AoI scheduler with a desirable throughput and energy constraint.

III. Minimum AAoI Scheduling Policy

At time-slot t, we define the state of the system as $\boldsymbol{S}[t] = [H[t], X[t], Q[t], \boldsymbol{W}[t], R[t], \tilde{G}[t]] \in \mathbb{S}$. The scheduling decision (or action) is to choose an energy $U[t] \in \mathbb{U}$ to transmit a packet. Choose $U[t] = 0$ when not to transmit a packet, or when $Q[t] = 0$. We note that the state space $\mathbb{S}$ and the action space $\mathbb{U}$ are finite sets. From (1)–(4), it is clear that the state $\boldsymbol{S}[t]$ follows a controlled Markov chain, with the state transition probability $\mathsf{P}(\boldsymbol{s}, \boldsymbol{u}, \boldsymbol{s}') = \mathbb{P}\{\boldsymbol{S}[t+1] = \boldsymbol{s}' | \boldsymbol{S}[t] = \boldsymbol{s}, U[t] = u\}$. At each time-slot t, for state $\boldsymbol{s} = [h, x, q, \boldsymbol{w}, r, \tilde{g}]$ and action u, the one stage cost is defined as $C(\boldsymbol{s}, u) = h$. The problem is to choose $\{U[t]\}$ such that AAoI is minimum subject to average energy and throughput constraints. Thus, the tuple $(\mathbb{S}, \mathbb{U}, \mathsf{P}, C)$ defines a Markov Decision Process (MDP).

Let $\pi = (\mu_0, \mu_1, \mu_2, \ldots) \in \Pi_{\mathrm{NA}}$ be a policy in the class of non-anticipated policies, i.e., μ_t depends on the history of past state-action pairs until time-slot $t-1$ and state $\boldsymbol{S}[t]$. Under policy π, the average cost of the system, which is AAoI, is

$$J^\pi(\boldsymbol{s}_0) := \lim_{T\to\infty} \frac{1}{T}\mathbb{E}_\pi\left[\sum_{t=0}^{T-1} C(\boldsymbol{S}[t], U[t]) \middle| \boldsymbol{S}[0] = \boldsymbol{s}_0\right], \quad (6a)$$

where $\boldsymbol{s}_0$ is the state of the system at time-slot 0. At time-slot t, the transmit-energy and throughput obtained are $U[t] =: k_1(\boldsymbol{S}[t], U[t])$ and $X[t] =: k_2(\boldsymbol{S}[t], U[t])$, respectively. The time-average transmit-energy and throughput are given by

$$K_1^\pi(\boldsymbol{s}_0) := \lim_{T\to\infty} \frac{1}{T}\mathbb{E}_\pi\left[\sum_{t=0}^{T-1} k_1(\boldsymbol{S}[t], U[t]) \middle| \boldsymbol{S}[0] = \boldsymbol{s}_0\right], \quad (6b)$$

$$K_2^\pi(\boldsymbol{s}_0) := \lim_{T\to\infty} \frac{1}{T}\mathbb{E}_\pi\left[\sum_{t=0}^{T-1} k_2(\boldsymbol{S}[t], U[t]) \middle| \boldsymbol{S}[0] = \boldsymbol{s}_0\right]. \quad (6c)$$

We are interested in the average transmit-energy $K_1^\pi(\boldsymbol{s}_0) \le \bar{e}$ and the throughput $K_2^\pi(\boldsymbol{s}_0) \ge \bar{\eta}$. Note that (6a)–(6c) do not depend on the initial state $\boldsymbol{s}_0$ [9, p. 276], and they define AAoI, average transmit-energy, and throughput, respectively. Thus, we define

Problem 1 (Constrained AAoI Scheduling):

$$\min_{\pi\in\Pi_{\mathrm{NA}}} \quad J^\pi(\boldsymbol{s}_0), \quad (7a)$$

$$\text{s.t.} \quad K_1^\pi(\boldsymbol{s}_0) \le \bar{e}, \quad (7b)$$

$$K_2^\pi(\boldsymbol{s}_0) \ge \bar{\eta}, \quad (7c)$$

where $\bar{e}$ is the available average transmit-energy and $\bar{\eta}$ is the desired throughput. Thus, we have an average cost constrained MDP (CMDP), which is represented by the tuple $(\mathbb{S}, \mathbb{U}, \mathsf{P}, C, K_1, K_2, \bar{e}, \bar{\eta})$.

Let $\pi^* \in \Pi_{\mathrm{NA}}$ be a solution to *Problem 1*. Since, the CMDP that defines *Problem 1* has finite state and action spaces, the optimal policy π^* need not be stationary deterministic [10], [11]. From [11, Th. 3.6], the CMDP can be formulated using Lagrangian relaxation, with the Lagrange multiplier $\boldsymbol{\lambda} = [\lambda_1, \lambda_2] \ge \boldsymbol{0}$. The one-stage Lagrangian for state $\boldsymbol{s} = [h, x, q, \boldsymbol{w}, r, \tilde{g}]$ and action u is

$$C_{\boldsymbol{\lambda}}(\boldsymbol{s}, u) = h + \lambda_1(u - \bar{e}) - \lambda_2(x - \bar{\eta}), \quad (8)$$

and under policy π, the average cost for Lagrangian multiplier $\boldsymbol{\lambda}$ is

$$J_{\boldsymbol{\lambda}}^\pi(\boldsymbol{s}_0) := \lim_{T\to\infty} \frac{1}{T}\mathbb{E}_\pi\left[\sum_{t=0}^{T-1} C_{\boldsymbol{\lambda}}(\boldsymbol{S}[t], U[t]) \middle| \boldsymbol{S}[0] = \boldsymbol{s}_0\right]. \quad (9)$$

The CMDP can be equivalently formulated as

$$\inf_{\pi\in\Pi_{\mathrm{NA}}} \sup_{\boldsymbol{\lambda}\ge 0} J_{\boldsymbol{\lambda}}^\pi(\boldsymbol{s}_0) = \sup\nolimits_{\boldsymbol{\lambda}\ge 0} \inf\nolimits_{\pi\in\Pi_{\mathrm{NA}}} J_{\boldsymbol{\lambda}}^\pi(\boldsymbol{s}_0), \quad (10)$$

where the order of inf and sup can be interchanged, [11, Th. 3.6]. The inner unconstrained scheduling problem is

Problem 2 (Unconstrained AAoI Scheduling):

$$J_{\boldsymbol{\lambda}}^* = \min_{\pi\in\Pi_{\mathrm{NA}}} J_{\boldsymbol{\lambda}}^\pi(\boldsymbol{s}_0), \quad (11a)$$

$$\text{and } \pi_{\boldsymbol{\lambda}}^* \in \arg\min_{\pi\in\Pi_{\mathrm{NA}}} J_{\boldsymbol{\lambda}}^\pi(\boldsymbol{s}_0). \quad (11b)$$

A policy π is called $\boldsymbol{\lambda}$-optimal if it achieves $J_{\boldsymbol{\lambda}}^*$. In *Problem 2*, we have an average cost MDP having finite state and action spaces, and hence, a stationary deterministic $\boldsymbol{\lambda}$-optimal policy $\pi_{\boldsymbol{\lambda}}^*$ exists, which is obtained by solving the Bellman equation [9, p. 276],

$$J_{\boldsymbol{\lambda}}^* + V_{\boldsymbol{\lambda}}^*(\boldsymbol{s}) = \min_{u\in\mathbb{U}}\left[C_{\boldsymbol{\lambda}}(\boldsymbol{s}, u) + \mathbb{E}\left[V_{\boldsymbol{\lambda}}^*(\boldsymbol{s}') \middle| \boldsymbol{s}, u\right]\right], \quad (12)$$

where $V_{\boldsymbol{\lambda}}^*$ is the value function, $\boldsymbol{s}'$ is the next state of the system when the current state, action is $\boldsymbol{s}$, u. $V_{\boldsymbol{\lambda}}^*$ and hence, the stationary policy $\pi_{\boldsymbol{\lambda}}^* = [\mu_{\boldsymbol{\lambda}}^*, \mu_{\boldsymbol{\lambda}}^*, \ldots]$ can be computed by a value iteration (VI) method [9, p. 304], and is given by

$$\begin{aligned} v_0(\boldsymbol{s}) &= 0, \\ v_{i+1}(\boldsymbol{s}) &= \min_{u\in\mathbb{U}}\left[C_{\boldsymbol{\lambda}}(\boldsymbol{s}, u) + \mathbb{E}\left[v_i(\boldsymbol{s}') \middle| \boldsymbol{s}, u\right]\right], \ i = 0, 1, \ldots, \\ \mu_{i+1}(\boldsymbol{s}) &= \arg\min_{u\in\mathbb{U}}\left[C_{\boldsymbol{\lambda}}(\boldsymbol{s}, u) + \mathbb{E}\left[v_i(\boldsymbol{s}') \middle| \boldsymbol{s}, u\right]\right], \\ \mu_{\boldsymbol{\lambda}}^*(\boldsymbol{s}) &= \lim_{i\to\infty} \mu_i(\boldsymbol{s}). \end{aligned} \quad (13)$$

A. AAoI Optimal Scheduler (AOPT)

When the available transmit-energy is increased, one can increase $U[t]$ in time-slots t to avoid a packet drop, thereby achieving a higher throughput. Thus, throughput increases monotonically with $\bar{e}$. Hence, to achieve a desired throughput of $\bar{\eta}$, we need an average transmit-energy of at least $\underline{e}(\bar{\eta})$. Thus, the constraints (7b), (7c) are jointly given by $\underline{e} \le K_1^{\pi}(\boldsymbol{s}_0) \le \bar{e}$. As λ_1 increases, $J_{\boldsymbol{\lambda}}^*$ increases, and $K_1^{\pi_{\lambda}^*}(\boldsymbol{s}_0)$ decreases. Thus, it is enough to look for $\boldsymbol{\lambda}^* = \inf\{\boldsymbol{\lambda} > \boldsymbol{0} : K_1^{\pi_{\lambda^*}^*}(\boldsymbol{s}_0) \le \bar{e}\}$. Thus, the optimal policy for *Problem 1* is given by

Theorem 1: For *Problem 1*, a stationary optimal policy π^* exists. π^* randomizes between stationary deterministic policies π_1^* and π_2^* with probabilities θ and $1-\theta$. The policies π_1^* and π_2^* are $\boldsymbol{\lambda}^*$-optimal policies for *Problem 2* for some $\boldsymbol{\lambda} = \boldsymbol{\lambda}^*$.

Proof: Let $K_{1,\lambda}$ be the average transmit-energy of λ-optimal policy. For a given $\bar{e}$, define λ^* as

$$\lambda^* = \inf\{\lambda > 0 : K_{1,\lambda} \le \bar{e}\}. \tag{14}$$

Since there exists a policy $f(\boldsymbol{s}) := 0, \forall \boldsymbol{s}$ such that $K_1^f(\boldsymbol{s}_0) \le \bar{e}$, from [10, Lemma 3.3], we have $\lambda^* < \infty$, thus making [10, Hypothesis 4.1] true. Hence, it follows from [10, Th. 4.4] that there exists an optimal stationary randomized policy for *Problem 1*. ∎

To find the optimal policy, we need λ^*, π_1^*, π_2^* and θ. λ^* is given by (14). For a given λ^*, consider an increasing sequence $\{\lambda_n\} \to \lambda^*$, and a decreasing sequence $\{\lambda_n'\} \to \lambda^*$. Let

$$\lim_{\lambda_n \uparrow \lambda^*} K_{1,\lambda_n} = \alpha_1, \lim_{\lambda_n' \downarrow \lambda^*} K_{1,\lambda_n'} = \alpha_2 \tag{15}$$

where $\alpha_2 \le \bar{e} \le \alpha_1$. It is shown in [10] that $\{\pi_{\lambda_n}^*\}$ converges to λ^*-optimal policy π_1^* and $\{\pi_{\lambda_n'}^*\}$ converges to λ^*-optimal policy π_2^*. Also, θ is chosen such that $K_1^{\pi^*}(\boldsymbol{s}_0) = \bar{e}$, i.e.,

$$\theta = \frac{\bar{e} - \alpha_2}{\alpha_1 - \alpha_2}. \tag{16}$$

Since, the optimal policy is computationally intensive, we propose a sub-optimal algorithm in Section III-B.

Effect of LCFS/FCFS on AOPT: At time-slot t, let the state be $\boldsymbol{S}[t] = \boldsymbol{s} = [h, x, q, \boldsymbol{w}, r, \tilde{g}]$, where $\boldsymbol{w} = [w_1, w_2, \ldots, w_q]$. Since $w_1 > w_2 > \cdots > w_q$, among packets in the buffer, transmitting the ToL packet (LCFS) results in lower AoI compared to transmitting the HoL packet (FCFS). Also, in LCFS, if the freshest packet received at time-slot t is generated at $z > t - w_q$, then $H[t] = (t-z)_M < w_q$, and hence, transmitting any packet in the buffer would not reduce AoI. Thus, for LCFS, 1-buffer is sufficient for optimal AoI. In FCFS, any packet in the transmit-queue is fresher than the freshest packet received, and thus, AOPT intends to transmit all packets in the buffer. Thus, FCFS has a higher throughput than LCFS.

Effect of $\bar{e}$ on AOPT: When $\bar{e}$ is small, large number of packet drops occur. The optimal policy, for small $H[t]$ is $U[t] = 0$, and for large $H[t]$ is $U[t] > 0$. Further, in LCFS, if there is a preemption of an arrival, energy spent for unsuccessful transmissions goes waste. Hence, the optimal policy chooses large $U[t]$ (when $H[t]$ is large), to effectively use transmit-energy, and deliver packets quickly. Thus, the performance of LCFS is almost the same for delay deadlines $d >= 1$.

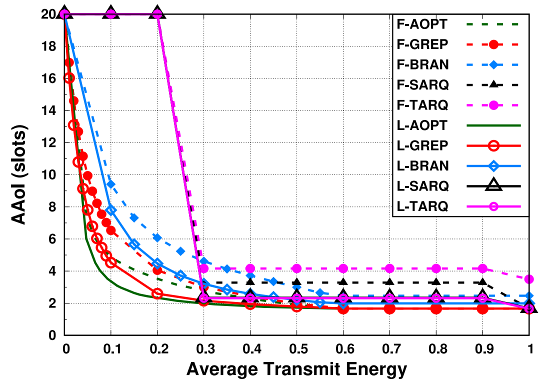

Fig. 3. AAoI vs average transmit-energy for $p = 0.6$, $e_1 = 1$, $e_2 = 0.3$, $p_G(1) = 0.4$, $p_G(2) = 0.6$, $d = 3$.

Algorithm 1 GREP

Input: $\boldsymbol{s}_t, \bar{e}, E_0 = 0, E_1, E_2, \cdots, E_K$
Output: u_t
if $\bar{e} \geqslant E_K$ **then**
  Choose action $u_t = f_K(\boldsymbol{s}_t)$
else if $E_{j-1} \leqslant \bar{e} < E_j$ for some $j = 1, 2, \ldots, K$ **then**
  Randomize between policies f_{j-1} and f_j
  Choose action $u_t = \begin{cases} f_j(\boldsymbol{s}_t), & \text{w.p. } \theta, \\ f_{j-1}(\boldsymbol{s}_t), & \text{w.p. } 1-\theta. \end{cases}$ where θ is chosen such that $\bar{e}$ is achieved.
end if
return u_t

Complexity of AOPT: In AOPT, each VI step requires $O(AS^2)$ multiplications and additions. Thus, in each iteration, the number of computations is $O(AS^2)$. If VI is performed m times, the complexity is $O(mAS^2)$, which is intensive, and hence, we propose the following simple policy.

B. Greedy Randomization of Energy Policy (GREP)

Define $e_0 = 0$. For any state $\boldsymbol{s} = [h, x, q, \boldsymbol{w}, r, \tilde{g}]$, define the deterministic policies $f_0, f_1, f_2, \ldots, f_K$, as follows.

$$f_i(\boldsymbol{s}) = \begin{cases} 0, & \text{if } q = 0, \\ e_i, & \text{if } q > 0. \end{cases}$$

From (6b), the average energy of policy f_i is $K_1^{f_i}(\boldsymbol{s}_0) =: E_i$. Note that $E_1 > E_2 \cdots > E_K$. We describe GREP in Algorithm 1.

Complexity of GREP: E_j, the average transmit-energy for policy f_j requires $O(1)$ computations. Comparison of $\bar{e}$ with $E_1, \ldots, E_K$ to find $E_{j-1} \le \bar{e} < E_j$ requires a maximum of $K = O(A)$ comparisons. Thus, GREP requires $O(A)$ computations, whereas AOPT requires $O(mAS^2)$ computations. Thus, the complexity of GREP is far less compared to that of AOPT.

IV. Numerical Results

We study the policies i) AOPT, ii) GREP, iii) BRAN,[4] iv) SARQ [2], and v) TARQ [6], under FCFS and LCFS disciplines, and denote each policy by F-policy and L-policy.

[4]BRAN policy chooses $U[t] \in \{0, e_1, e_2, \ldots, e_K\}$ with a fixed probability distribution such that average transmit-energy is $\bar{e}$.

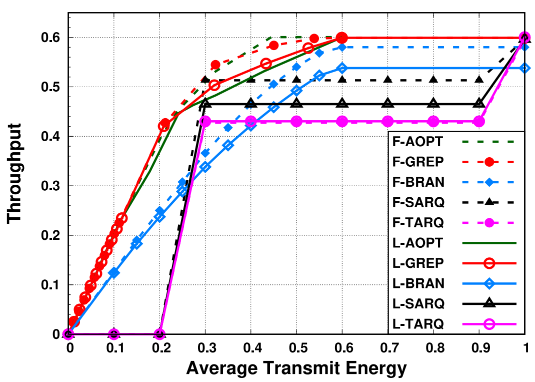

Fig. 4. Throughput vs average transmit-energy for $p = 0.6$, $e_1 = 1, e_2 = 0.3$, $p_G(1) = 0.4, p_G(2) = 0.6$, $d = 3$.

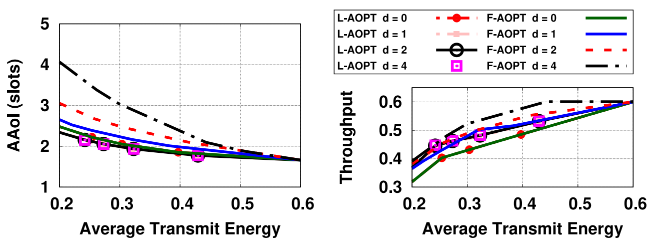

Fig. 5. AAoI and Throughput vs average transmit-energy, for $d \in \{0, 1, 2, 4\}$, $p = 0.6$, $e_1 = 1, e_2 = 0.3$, $p_G(1) = 0.4, p_G(2) = 0.6$.

We consider the following parameters: Bernoulli(p) arrivals with $p = 0.6$, $K = 2$ channel states, energy levels $e_1 = 1.0$, $e_2 = 0.3$, and $p_G(1) = 0.4$, $p_G(2) = 0.6$. The energy levels are normalized such that $e_1 = 1.0$, and hence, it is enough to consider $\bar{e} \leq 1$.

In Fig. 3, AAoI is plotted against average transmit-energy, $\bar{e}$. For any $\bar{e}$, LCFS achieves lower AAoI than FCFS. As $\bar{e}$ increases, AAoI decreases due to more early successes and less retransmissions/drops. Also, AOPT policy achieves the smallest AAoI for any $\bar{e}$, and GREP closely matches AOPT. BRAN follows the same trend as AOPT, but with poor performance for $\bar{e} < 0.2$. SARQ/TARQ achieves an AAoI of $M = 20$ for $\bar{e} < e_2$ as SARQ/TARQ do not combine the previous transmissions and $\bar{e} < e_2$. For any delay deadline d, the same trend is observed, but is shown for $d = 3$.

In Fig. 4, throughput is plotted versus $\bar{e}$. For any $\bar{e}$, FCFS achieves higher throughput than LCFS, as in LCFS, packet drops can be more due to preemption. For large $\bar{e}$, a transmission is successful with high probability resulting in low drops, and high throughput. For any $\bar{e}$, AOPT achieves the largest throughput, and GREP performs very close to AOPT. For SARQ/TARQ, when $\bar{e} < e_2$, there is no successful transmission in any slot, and hence, throughput is zero.

In Fig. 5, for AOPT, we plot AAoI and throughput versus $\bar{e}$ for $d \in \{0, 1, 2, 4\}$. For FCFS, as d increases, both AAoI and throughput increases. For LCFS, it is always beneficial to have as much transmit-energy at arrival epochs (at which $H[t]$ is large for small $\bar{e}$, or even small $H[t]$ for large $\bar{e}$) as preemption by a new arrival will incur a loss in energy. Thus, for $d > 1$, L-AOPT AAoI and throughput saturates. In Fig. 6, AAoI is plotted against throughput. A large throughput requires large $\bar{e}$, and hence a small AAoI, and vice versa.

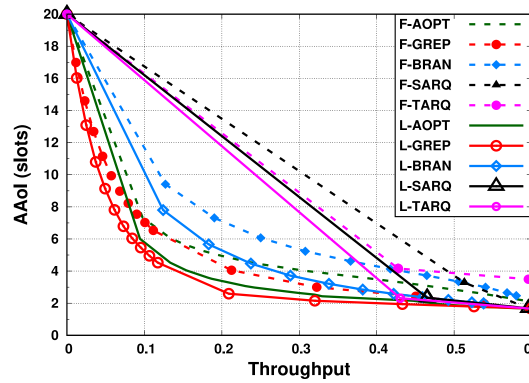

Fig. 6. AAoI vs throughput for $p = 0.6$, $e_1 = 1, e_2 = 0.3$, $p_G(1) = 0.4, p_G(2) = 0.6$, $d = 3$.

V. Conclusion

We have investigated the minimum age scheduling problem with HARQ in a fading link with delayed CSIT, packet delay deadlines, and transmit-energy and throughput constraints. We formulate the problem as a CMDP, and show that the optimum policy (AOPT) is a randomization of stationary deterministic policies. We propose GREP, a simple policy that randomizes between two adjacent energy levels e_{j-1}, e_j, and BRAN, a policy that blindly randomizes across all transmit-energy levels. We compare the AAoI and throughput performance of all the proposed policies with SARQ and TARQ. We show that GREP performs very close to that of the optimal policy. A future direction is to extend this letter to multiple sources.

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